CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations March 2023

Programme	: Integrated B.Sc.-M.Sc.	Session	: 2022-2023
Semester	: First	Max. Time	: 3 Hours
Course Title	: Introductory Calculus and Analysis	Max. Marks	: 70
Course Code	: SBSMAT 03 01 01 GE 5106		

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) If $y = \sin(m\sin^{-1}x)$; prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$.

   (b) Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P_0(1,1,0)$ in the direction of $\vec{v} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$.

   (c) Examine the convergence of the following integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

   (d) Use fundamental theorem to compute $\int_0^{\frac{\pi}{3}} \cos x \, dx$.

   (e) Determine the radius and interval of convergence of the following power series

   $$\sum_{n=1}^{\infty} \frac{6^n(4x-1)^{n-1}}{n}.$$

   (f) Find $\frac{dw}{dt}$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

   (g) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ and hence evaluate the same.

2. (a) If $y = e^{m\cos^{-1}x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ and find the value of $y_n(0)$.

   (b) Discuss the continuity and differentiability of the function

   $$f(x) = |x-1| + |x-2|$$

   in the interval $[0,3]$.

   (c) State and prove Lagrange's Mean Value Theorem.

3. (a) State and prove Darboux theorem.

   (b) Discuss the convergence of Gamma function.

   (c) Find the volume of the sphere $x^2 + y^2 + z^2 \leq a^2$, $a > 0$.

4. (a) Discuss the convergence of the series: $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \frac{5^5x^5}{5!} + \cdots$

   (b) Test the convergence and absolute convergence of the series

   $$\sum_{n=1}^{\infty}(-1)^{n+1}(\sqrt{n^2+1} - n).$$

   (c) Using Integral test, show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $0 < p \leq 1$.

5. (a) If $u = \log \frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.

(b) Find the maximum and minimum values of the function $f(x, y) = 3x+4y$ on the circle $x^2+y^2 = 1$.

(c) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that

$$(grad\, u).[(grad\, v) \times (grad\, w)] = 0.$$

6. (a) Show that $r^n\vec{r}$ is irrotational where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $|\vec{r}| = r$.

(b) Evaluate by Stoke's theorem $\oint_C (e^x\, dx + 2y\, dy - dz)$, where C is the curve $x^2 + y^2 = 4$, $z = 2$.

(c) Evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where $\vec{f} = (x + y^2)\hat{\imath} - 2x\hat{\jmath} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations March 2023

Programme: PG | **Session: 2022-23**

Semester: 1st | **Max. Time: 3 Hours**

Course Title: Mathematics for Chemists | **Max. Marks: 70**

Course Code: SBSMAT 01 01 02 GEC 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Find the polar coordinates (r, θ) of the point whose Cartesian coordinates are (x, y) = (3, 4).

b) Define Hermitian and Skew-Hermitian matrices with and example.

c) Show that $y = e^{-x} + 1$ is a solution of the first-order differential equation:

$$\frac{dy}{dx} + y - 1 = 0$$

d) Find the general solution of the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

e) Define singular point of differential equation and their types.

f) Verify that the function $f(x,t) = 3x^2 - 2xvt = 3v^2t^2$ is solution of partial differential equation $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$.

g) Find the probability of getting sum of 8 and 10 from two throws of a die.

Q 2. (2X7=14)

a) Rotate the square whose corners have positions in the xy-plane: (x, y) = (2,1), (3,1), (3,2), (2,2) through '$\pi/6$' about the origin.

b) Evaluate the determinant of 'Adj (A)' i.e. |Adj (A)|, where, A= $\begin{bmatrix} 3 & 2 & -1 \\ 6 & 1 & 5 \\ -9 & 3 & 4 \end{bmatrix}$.

c) Sketch the graphs of e^{2x} and e^{-2x} for values $-1.5 \le x \le 1.5$.

Q3. (2X7=14)

a) The reversible reaction A⇌B, first order in both directions, has rate equation:

$$\frac{dx}{dt} = k_1(a - x) - k_{-1}\,x$$

Find x(t) for initial condition x(0) = 0.

b) Find the general solution of differential equation:

$$\frac{dy}{dx}+\frac{2y}{x}=2\cos x$$

c) Solve the initial value problem:

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0,\quad y(0)=0,\quad y'(0)=5.$$

Q 4. (2X7=14)

a) Use the power-series method to solve the equation

$$\frac{d^2y}{dx^2}+y=0.$$

b) Evaluate the line integral $I=\int_C \boldsymbol{a}.\boldsymbol{dr}$, where $\boldsymbol{a}=(x+y)i+(y-x)j$, along $y=x^2$ from (1, 1) to (4, 2) in the xy −plane.

c) Find $y=\int(2x^4+3x^2+100x)\,dx$ subject to conditions (i) $(x,y)=(2,0)$, (ii) $y=(3,10)$.

Q 5. (2X7=14)

a) Find the probability of throwing (i) at least 2 'sixes' (ii) at most 1 'two' (iii) at most 3 'four' in 5 throws of a fair die.

b) Find the mean, median, mode and range for the given data: 90, 94, 53, 68, 79, 94, 53, 65, 87, 90, 70, 69, 65, 89, 85, 53, 47, 61, 27, 80.

c) X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find (i) $P(x < 40)$ (ii) $P(x > 21)$ (iii) $P(30 < x < 35)$.

CENTRAL UNIVERSITY OF HARYANA

Term End Semester Examinations March 2023

Program: M.Sc. Mathematics **Session: 2022-23**

Semester: First **Max. Time: 3 Hours**

Course Title: Differential Equations **Max. Marks: 70**

Course Code: SBSMAT 01 01 04 C 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Show that e^x *and* e^{3x} are linearly independent solutions of $y'' - 5y' + 6y = 0$. Find the solution y(x) with the property that y(0) = 0 and $y'(0) = 1$.

b) Show that the system of confocal conics $\frac{x^2}{(a^2+\mu)} + \frac{y^2}{(b^2+\mu)} = 1$ is self orthogonal. Here μ is a parameter.

c) Find the solution of $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ using Lagrange method.

d) Solve the following equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

e) Write down the canonical form of one dimensional wave equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$.

f) Define singular solution. Find general and singular solution of $9p^2(2-y)^2 = 4(3-y)$.

g) Find solution in generalized series form about x =0 of the differential equation

$$3x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$$

Q 2. (2X7=14)

a) Find the solution of the system of ODE.

$$\frac{dx}{dt} = x(1-2y)$$

$$\frac{dy}{dt} = -y(1-3x).$$

b) State and prove the Picard Existence and uniqueness theorem. Whether the differential equation $\frac{dy}{dx} = \frac{y}{x}$ with initial condition $y(1) = 0$ have unique solution or infinite solutions, Discuss.

c) Show that the functions $1-x, 1-2x+\frac{x^2}{2}$ and $1-3x+\frac{3x^2}{2}-\frac{x^3}{6}$ are orthogonal with respect to e^{-x} on $0 \le x < \infty$. Determine the corresponding orthnormal functions.

Q3. (2X7=14)

a) Find the Eigen values of the differential equation $\frac{d^2y}{dx^2} + \lambda y = 0$ with boundary conditions $y(0) = 0, y(\pi) = 0$.

b) Show that $(1 - 2xz + z^2)^{-\frac{1}{2}} = \sum_0^\infty z^n P_n(x), |x| \leq 1, |z| < 1$. Here $P_n(x)$ is Legendre polynomial of degree n.

c) Solve $xy'' + 2y' + \frac{xy}{2} = 0$ in terms of Bessel's functions.

Q 4. (2X7=14)

a) Find the complete integral of the equation $p^2x + q^2y = z$ using Charpit's method.

b) Find the complete integral of $p_1p_2p_3 = z^3x_1x_2x_3$. Here $p_i = \frac{\partial z}{\partial x_i}; i = 1,2,3.$

c) Solve the Cauchy problem: $2p^2x + qy - u = 0$ with Cauchy data on x-axis $u(x, 1) = -\frac{1}{2}x$.

Q 5. (2X7=14)

a) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x, 0) = 3\sin(n\pi x), u(0, t) = 0$ and $u(1, t) = 0$, where $0 < x < 1, t > 0$.

b) Find the deflection u(x, y, t) of the square membrane with $a = b = c = 1$, if the initial velocity is zero and the initial deflection is $f(x, y) = A\, sin(\pi x)\sin(2\pi y)$.

c) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions:

$$u(0, y) = u(l, y) = 0, u(x, 0) = 0 \text{ and } u(x, a) = \sin(\frac{n\pi x}{l}).$$

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations March-2023

Programme	: Integrated B.Sc.-M.Sc. Mathematics	Session	: 2022-23
Semester	: First	Max. Time	: 3 Hours
Course Title	: Algebra and Geometry	Max. Marks	: 70
Course Code	: SBSMAT 03 01 02 C 5106		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) State De Moivre's theorem.
   (b) Explain Synthetic division.
   (c) Suppose that the vectors u, v and w are linearly independent. Show that the vectors $u+v, u-v, u-2v+w$ are also linearly independent.
   (d) Explain Euclidean algorithm.
   (e) Find the equation to the sphere whose centre is $(2, -3, 4)$ and radius is 5.
   (f) Explain right circular cone.
   (g) Define ruled surfaces.

2. (a) Find the condition that the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ is equal to zero.
   (b) Solve the equation $x^3 - 12x - 65 = 0$ by Cardan's method.
   (c) State and prove fundamental of algebra. If the roots of the equation $x^n - 1 = 0$ be $1, \alpha, \beta, \gamma, \cdots$ show that $(1-\alpha)(1-\beta)(1-\gamma)\cdots = n$.

3. (a) Prove that the relation R defined on the set of positive integers $(x, y) \in R$ if $x - y$ is divisible by 5 is an equivalence relation.
   (b) State and prove Fermat's little theorem.
   (c) Prove that the unit interval $[0, 1]$ is uncountable.

4. (a) State and prove Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
   (b) Let $B = \{(1,1,1), (1,1,0), (1,0,0)\}$ be an ordered basis of R^3. Find the matrix of the linear transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = \{x + y + z, x + y, x\}$.
   (c) Find the values of λ and μ for which $x + y + 2z = 3,\ \ 2x - y + 3z = 4,\ \ 5x - y + \lambda = \mu$ have (i) an unique solution (ii) many solutions (iii) no solution.

5. (a) Find the equation of the plane which contains the line of intersection of the planes $x+2y+3z-4 = 0$ and $2x + y - z + 5 = 0$ and which is perpendicular to the plane $5x + 3y - 6z + 8 = 0$.

(b) Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}; \quad \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}.$$

Show also that the equations of the shortest distance are $11x+2y-7z+6=0=7x+y-5z+7$.

(c) Obtain the equations of the spheres which pass trough the circle $x^2+y^2+z^2=5$, $x+2y+3z=3$ and touch the plane $4x+3y=15$.

6. (a) Find the condition for the general equation of the second degree to represent a cone and find the co-ordinates of its vertex.

(b) Obtain the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1$, $z=3$.

(c) Find the locus of the equal conjugate diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations March-2023

Programme	: Integrated B.Sc.-M.Sc. Mathematics	Session	: 2022-23
Semester	: First	Max. Time	: 3 Hours
Course Title	: Algebra and Geometry	Max. Marks	: 70
Course Code	: SBSMAT 03 01 02 C 5106		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) State De Moivre's theorem.
   (b) Explain Synthetic division.
   (c) Suppose that the vectors u, v and w are linearly independent. Show that the vectors $u+v, u-v, u-2v+w$ are also linearly independent.
   (d) Explain Euclidean algorithm.
   (e) Find the equation to the sphere whose centre is $(2,-3,4)$ and radius is 5.
   (f) Explain right circular cone.
   (g) Define ruled surfaces.
2. (a) Find the condition that the sum of two roots of the equation $x^4+px^3+qx^2+rx+s=0$ is equal to zero.
   (b) Solve the equation $x^3-12x-65=0$ by Cardan's method.
   (c) State and prove fundamental of algebra. If the roots of the equation $x^n-1=0$ be $1, \alpha, \beta, \gamma, \cdots$ show that $(1-\alpha)(1-\beta)(1-\gamma)\cdots = n$.
3. (a) Prove that the relation R defined on the set of positive integers $(x, y) \in R$ if $x-y$ is divisible by 5 is an equivalence relation.
   (b) State and prove Fermat's little theorem.
   (c) Prove that the unit interval $[0,1]$ is uncountable.
4. (a) State and prove Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix
   $$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$
   (b) Let $B = \{(1,1,1), (1,1,0), (1,0,0)\}$ be an ordered basis of R^3. Find the matrix of the linear transformation $T: R^3 \to R^3$ defined by $T(x,y,z) = \{x+y+z, x+y, x\}$.
   (c) Find the values of λ and μ for which $x+y+2z=3,\ \ 2x-y+3z=4,\ \ 5x-y+\lambda=\mu$ have (i) an unique solution (ii) many solutions (iii) no solution.
5. (a) Find the equation of the plane which contains the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and which is perpendicular to the plane $5x+3y-6z+8=0$.

(b) Find the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-4}{4};\ \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}.$$

Show also that the equations of the shortest distance are $11x+2y-7z+6=0=7x+y-5z+7$.

(c) Obtain the equations of the spheres which pass trough the circle $x^2+y^2+z^2=5,\ x+2y+3z=3$ and touch the plane $4x+3y=15$.

6. (a) Find the condition for the general equation of the second degree to represent a cone and find the co-ordinates of its vertex.

(b) Obtain the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1,\ z=3$.

(c) Find the locus of the equal conjugate diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations March-2023

Programme	: M.Sc. Mathematics	Session	: 2022-23
Semester	: First	Max. Time	: 3 Hours
Course Title	: Complex Analysis	Max. Marks	: 70
Course Code	: SBSMAT 01 01 03 C 3104		

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Find the general value of $\log(\sqrt{3}-i)$.

   (b) Show that $e^{(\frac{2+\Pi i}{4})} = \sqrt{\frac{e}{2}}(1+i)$.

   (c) let $f(z) = u + iv$ be an analytic function in a domain D. Prove that $f(z)$ is constant in D if $|f(z)|$ is constant.

   (d) Determine the number of zeros of the equation $2z^5 - 6z^2 + z + 1 = 0$ in the annulus $1 \leq |z| < 2$.

   (e) Investigate the behaviour of the series $\sum_{n=0}^{\infty} \frac{z^{4n}}{4n+1}$ on the circle of convergence.

   (f) Find the residue of the function $f(z) = \frac{(logz)^3}{(z^2+1)}$ at $z = i$.

   (g) Find the fixed point and normal form of the bilinear transformation: $w = \frac{3iz+1}{z+i}$.

2. (a) Show that the function $f(z) = \begin{cases} \frac{zRez}{|z|} & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous in the entire complex plane.

   (b) Prove that $u = y^3 - 3x^2y$ is a harmonic function and find its harmonic conjugate. Also find the analytic function $f(z)$ in terms of z.

   (c) Give an example of a function which not analytic at any point, even though the Cauchy-Riemann equations are satisfied thereat.

3. (a) Evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where C is the Circle $|z| = 4$.

   (b) Prove that every bounded entire function f in the complex plane is constant throughout the plane.

   (c) State and prove Morera's Theorem.

4. (a) Find the Laurent or Taylor series expension of $\frac{z}{(z-1)(z-3)}$ for $0 < |z-1| < 2$.

   (b) Find the residue at $z = \infty$ of the function $\sqrt{(z-\alpha)(z-\beta)}$.

   (c) State and prove argument principle.

5. (a) State and prove Maximum modulus theorem.

   (b) Show that a bilinear transformation two inverse point with respect to a circle or line onto inverse Point with respect to the image circle or image line.

   (c) Find the fixed point and the normal form of following bilinear transformation (i) $w = \frac{3z-4}{z-1}$ (ii) $w = \frac{z}{z-2}$.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations March-2023

Programme	: M.Sc. Mathematics	Session	: 2022-23
Semester	: First	Max. Time	: 3 Hours
Course Title	: Complex Analysis	Max. Marks	: 70
Course Code	: SBSMAT 01 01 03 C 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Find the general value of $\log(\sqrt{3}-i)$.
   (b) Show that $e^{(\frac{2+\Pi i}{4})} = \sqrt{\frac{e}{2}}(1+i)$.
   (c) let $f(z) = u + iv$ be an analytic function in a domain D. Prove that $f(z)$ is constant in D if $|f(z)|$ is constant.
   (d) Determine the number of zeros of the equation $2z^5 - 6z^2 + z + 1 = 0$ in the annulus $1 \leq |z| < 2$.
   (e) Investigate the behaviour of the series $\sum_{n=0}^{\infty} \frac{z^{4n}}{4n+1}$ on the circle of convergence.
   (f) Find the residue of the function $f(z) = \frac{(logz)^3}{(z^2+1)}$ at $z = i$.
   (g) Find the fixed point and normal form of the bilinear transformation: $w = \frac{3iz+1}{z+i}$.

2. (a) Show that the function $f(z) = \begin{cases} \frac{zRez}{|z|} & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous in the entire complex plane.
   (b) Prove that $u = y^3 - 3x^2y$ is a harmonic function and find its harmonic conjugate. Also find the analytic function $f(z)$ in terms of z.
   (c) Give an example of a function which not analytic at any point, even though the Cauchy-Riemann equations are satisfied thereat.

3. (a) Evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where C is the Circle $|z| = 4$.
   (b) Prove that every bounded entire function f in the complex plane is constant throughout the plane.
   (c) State and prove Morera's Theorem.

4. (a) Find the Laurent or Taylor series expension of $\frac{z}{(z-1)(z-3)}$ for $0 < |z-1| < 2$.
   (b) Find the residue at $z = \infty$ of the function $\sqrt{(z-\alpha)(z-\beta)}$.
   (c) State and prove argument principle.

5. (a) State and prove Maximum modulus theorem.
   (b) Show that a bilinear transformation two inverse point with respect to a circle or line onto inverse Point with respect to the image circle or image line.
   (c) Find the fixed point and the normal form of following bilinear transformation (i) $w = \frac{3z-4}{z-1}$ (ii) $w = \frac{z}{z-2}$.

CENTRAL UNIVERSITY OF HARYANA

Term End Examination, March 2023

Programme: Ph.D. Mathematics	**Session: 2022-23**
Semester: Course work	**Max. Time: 3 Hours**
Course Title: Topics in Applied Mathematics	**Max. Marks: 60**
Course Code: SBS MAT 02 01 02 E 5106	

Instruction: Attempt any five questions out of the following. Each question carries equal marks.

Q1. Derive linear shape functions for a triangular element.

Q2. Consider the heat flow equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in the region $\square = [0 \le x \le 1] \times [t \ge 0]$ subject to the following initial and boundary conditions

$$u(x,0) = \begin{cases} x & 0 \le x \le 1/2 \\ 1-x & 1/2 \le x \le 1 \end{cases}$$

$$u(0,t) = u(1,t) = 0$$

Use Finite Element Method to evaluate the pivotal values at first time level by taking the nodes 0, 0.2, 0.5, 0.8 and 1. Take $\Delta t = 0.0625$. Also compute the flux at the end points $x = 0$ and $x = 1$.

Q3. Find an approximate solution to the given differential equation by Galerkin's method:

$$\frac{d^2u}{dx^2} - u = x; \quad 0 \le x \le 1$$

$$u(0) = 1, \; u(1) = 3$$

Use basis functions $\phi_1(x) = x(x-1)$ and $\phi_2(x) = x^2(x-1)$.

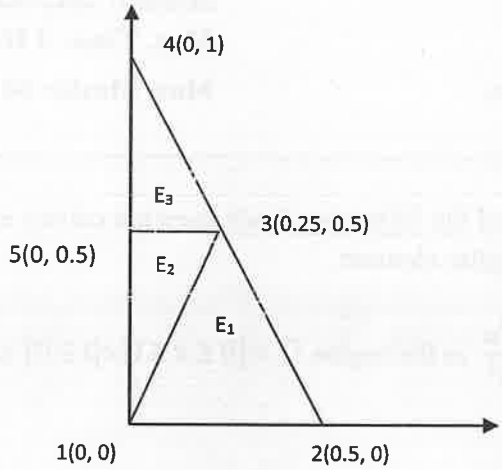
Q4. Solve the Poisson equation $\nabla^2 u = 3(x+y)$ by Finite Element method in the triangle enclosed by the axes and a straight line $2x + y = 1$. The boundary conditions on the three sides are as given below:

i) x-axis, $\frac{\partial u}{\partial y} = -x$.

ii) y-axis, $\frac{\partial u}{\partial x} = -y$.

iii) $2x + y = 1$, $u = x + y$.

The domain has been subdivided into three triangular elements as follows:[1]



Q5. What is the main advantage of using revised Simplex method. Use revised Simplex method to solve the following:

Maximize $Z = 6x + 3y + 4z - 2u + v$

Subject to $2x + 3y + 3z + u = 10$

$x + 2y + z + v = 8$

$x, y, z, u, v \geq 0$

Q6. Solve the given transportation problem for the minimum transportation cost. Also find the alternate optimal solution, if any.

$$\begin{array}{ccccccc} & & A & B & C & Supply \\ & X & 4 & 8 & 8 & 76 \\ From & Y & 16 & 24 & 16 & 82 \\ & Z & 8 & 16 & 24 & 77 \\ & Demand & 72 & 102 & 41 & \end{array}$$

Q7. Write 3×3 Fourier matrix. Find the Fourier transform of

$$f(x) = \left(1 - \frac{|x|}{a}\right) H\left(1 - \frac{|x|}{a}\right)$$

where $H(x)$ is the Heaviside unit step function.

Q8. Find the first order Hankel transform of

(a) e^{-ar} (b) $\frac{e^{-ar}}{r}$ (c) $\frac{\sin ar}{r}$

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations March 2023

Programme: B. Sc. – M.Sc. **Session: 2022-23**

Semester: 4th **Max. Time: 3 Hours**

Course Title: Mechanics **Max. Marks: 70**

Course Code: SBSMAT 03 04 01 C 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries 2 Marks.

2. Question no. 2 to 6 have three parts and students are required to answer any two parts of each question. Each part carries six marks.

Q 1. (2X5=10)

a) Forces $P-Q,\ P,\ P+Q$ act along the sides AB, BC, CA of an equilateral triangle. Find the magnitude, direction and line of action of resultant.

b) Find the centroid of the area of the parabola $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$ between the curve and the axes.

c) Derive the equations of equilibrium for common catenary.

d) The maximum velocity of a body moving in S.H.M. is 2 unit/sec and its period is 2 sec. What is its amplitude.

e) Prove that acceleration of a point moving in a curve with uniform speed is $\rho\left(\frac{d\psi}{dt}\right)^2$.

f) A particle moves in a plane under a central force which varies inversely as square of the distance from the fixed point. Find orbit.

g) To a man walking at 4 km/hr rain appears to fall vertically. If its actual velocity is 8 km/hr, find the actual acceleration.

Q 2. (2X6=12)

a) State and prove Varignon's theorem.

b) Forces of magnitude 1, 2, 3, 4, $2\sqrt{2}$ act respectively along the sides AB, BC, CD, DA and diagonal AC of a square ABCD of side a. Show that their resultant is a couple and its moment.

c) A uniform beam AB, 17m long whose mass is 120 kg rests with one end against a smooth vertical wall and the other end on a smooth horizontal floor, this end being tied by a chord 8 m long to a peg at the bottom of the wall. Find the tension of the chord.

Q3. (2X6=12)

a) Find the Centre of gravity of a segment of a circle.

b) Find the position of the Centre of gravity of the cycloid $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ which lies in the positive quadrant.

c) A given length 2s of a uniform chain has to be hung between two points at the same level and the tension has not to be exceed the weight of the length of the chain. Show that the greatest span is $\sqrt{b^2 + s^2} \log\left(\frac{b+s}{b-s}\right)$.

Q 4. (2X6=12)

a) A point executes a S.H.M. such that in two of its positions the velocities are u, v and the corresponding accelerations are α, β. Show that

i. the distance between the two points is $\frac{u^2-v^2}{\alpha-\beta}$

ii. the period of motion is $2\pi\sqrt{\frac{u^2-v^2}{\beta^2-\alpha^2}}$.

b) An elastic string of natural length l and modulus of elasticity λ has one end fixed at a point O on a smooth horizontal table. A particle of mass m is attached to the other end A pulled to the position B, where $AB = a$ and the length let go. Discuss the motion.

c) Show that the time of descent to the centre of force, the force varying inversely as the square of the distance from the centre through the first half of its initial distance is to that through the last half as $\pi + 2 : \mu - 2$.

Q 5. (2X6=12)

a) A particle describes a curve $r = ae^{\theta\cos\theta}$ with a constant velocity. Find the components of the velocity and acceleration along the radius vector and perpendicular to it.

b) A particle moves in a plane in such a manner that the tangential and normal acceleration are always equal and its velocity varies as $\exp\left(\tan^{-1}\frac{s}{c}\right)$, s being the length of the arc of the curve measured from a fixed point. Find the path.

c) A person travelling towards north-east, finds that the wind appears to blow from the north but when he doubled his speed, it seems to come from a direction inclined at an angle $\cot^{-1} 2$ from the east of north. Find the direction of the wind.

Q 6. (2X6=12)

a) Derive differential equation of central orbit in polar form.

b) A particle describes the equiangular spiral $r = ae^{\theta\cot\alpha}$ under a force to the pole. Find the law of force.

c) A particle moves with a central acceleration $\mu\left(r + \frac{a^4}{r^3}\right)$, being projected from an apse at a distance 'a' with the velocity $2a\sqrt{\mu}$. Prove that it describes the curve $r^2(2 + \cos\sqrt{3}\theta) = 3a^2$.

CENTRAL UNIVERSITY OF HARYANA

First Semester Term End Examinations March 2023

Programme: M.Sc. Mathematics
Semester: First
Course Title: Algebra-I
Course Code: SBSMAT 01 01 02 C 3104

Session: 2022-23
Max. Time: 3 Hours
Max. Marks: 70

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Questions no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks

Q 1. (4X3.5=14)

a) Prove that $(Q, +)$ is not cyclic group.
b) Find the permutation group isomorphic to the group $G = \{1, -1, i, -i\}$.
c) Define the class equation of a group with an example.
d) Define p −group with an example.
e) Define ring with an example.
f) Define ring homomorphism with an example.
g) Find the sum and product of the polynomials

$$f(x) = 4x - 5, g(x) = 2x^2 - 4x + 2 \text{ in } Z_8[x].$$

Q 2. (2X7=14)

a) If H and K are two subgroups of a group G, then HK is a subgroup of G iff $HK = KH$.
b) State and prove Cayley's Theorem.
c) If f is a homomorphism of G onto G' with kernel K, then $\frac{G}{K} \approx G'$.

Q3. (2X7=14)

a) State and Prove the First Sylow theorem.
b) Prove that any group of order 15 is cyclic.
c) Find all abelian and non-abelian groups of order 6.

Q 4. (2X7=14)

(a) Prove that $R = \{x + y\sqrt{2}, : x, y \in Q\}$ is a commutative ring.
(b) (i) Prove that, if X and Y are two ideals of a ring R then $X + Y$ is an ideal of R, containing both X and Y.
(ii) Define the prime ideal with an example.
(c) (i) Show that $f: Z_2 \longrightarrow Z_2, f(n) = n^2 - n$ is a ring homomorphism.
(ii) Prove that, if $f: R \longrightarrow R'$, is a homomorphism, then $Kerf = \{0\}$ iff f is one to one.

Q 5. (2X7=14)

a) (i) Show that $x^3 - 2$ is irreducible over Q.
(ii) Prove that $Z[\sqrt{-6}]$ is not a unique factorization domain.
b) Prove that, in a principal ideal domain, an element is prime iff it is irreducible.
c) Prove that $Z[i]$ is a Euclidian domain.

CENTRAL UNIVERSITY OF HARYANA

End Semester Examinations March 2023

Programme: M.Sc. Mathematics **Session:** 2022-23

Semester: First **Max. Time:** 3 Hours

Course Title: Programming in C **Max. Marks: 70**

Course Code: SBSMAT 01 01 05 C 3104

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and a half Marks.

2. Questions no. 2 to 5 have three parts and students need to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Classify procedural language with examples.

b) Write down any 7 backslash character constants with their meanings.

c) Consider the following code segment

```
int x = 6, y = 5, z = -3;

z = ( x = = 6 | | + +y)
```

Compute the values of x, y and z at the end of this segment.

d) Explain getchar() statement with an example.

e) What would be the output of the following program?

```
#include<stdio.h>
void main( )
{ int a[5]= {1, 2};
printf("\n %d, %d, %d\n", a[2], a[3], a[4]); }
```

f) Explain use of *break* and *continue* statements.

g) Determine the output of the following program?

```
#include<stdio.h>
void  main( )
{           f( );
            f( );
            f(  );
}
        void f(void)
        {           static int x=0;
                    printf("%d ", ++x);
        }
```

P.T.O.

Q 2. (2X7=14)

a) Explain all sections of basic structure of C-Programme.

b) Write a short note on the following

i) symbolic constants

ii) %f %e %g

iii) typedef

iv) enumerated data types

c) I) Which of following is not a correct variable name and why?

i) Price Mango   ii) sizeof   iii) Rajesh@gmail   iv) int_type

II) Find errors, if any, in the following declaration statement:

float letter, DIGIT;   double=p,q   exponent alpha, beta;

short char c;   long int m; count;   long float temp;

Q3. (2X7=14)

a) Write C expression equivalent to the following algebraic expressions and write a C program to compute these expressions:

i) $x+\frac{y^{-z}}{k}-\sqrt{x}$ (ii) $x^3-\frac{y^7}{-z}+e^x$

b) Write all steps and explain the output of the following program

```
#include<stdio.h>
main()
{
    int z;
    z=+4*5.0/2= =7&&(5!=3||3>=5)&&~0;
    printf("%d",z);
}
```

c) Write a short note on the following

i) Bitwise operators

ii) *isalpha*

iii) scanf("%d %*d %d", &a, &b, &c);

iv) printf("%-10d, %7.2f, %15s",a, b, c);

P.T.O.

Q 4. (2X7=14)

a) Write a short note on the following

i) *do while* statement

ii) *for(; ;)* loop

iii) *switch* statement

iv) nested *if else* statement

b) Write a program to print a pyramid of numbers with n rows. An example of the pyramid with 5 rows is shown below

```
        1
      2 3 2
    3 4 5 4 3
  4 5 6 7 6 5 4
5 6 7 8 9 8 7 6 5
```

c) Use a two-dimensional array to write a C- program for the multiplication of two matrices A_{mxn} and B_{pxq}.

Q 5. (2X7=14)

a) I) What will be the output if we compile and execute the following c code?

```
void main()
{ int a=10,*j;
int *k;
j=k=&a;
printf("%d\t%d\t%d",a,*j,*k); }
```

II) Write a short note on the following

i) *strcpy* ( ) function

ii) *return* statement

iii) * and & operators

b) Explain function definition, call and declaration with the help of a C- programme.

c) Write a program to print the addresses and values of a two dimensional array.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations March 2023

Programme: B. Sc. – M.Sc. **Session: 2022-23**

Semester: 1st **Max. Time: 3 Hours**

Course Title: Calculus **Max. Marks: 70**

Course Code: SBSMAT 03 01 01 C 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries 2 Marks.

2. Question no. 2 to 6 have three parts and students are required to answer any two parts of each question. Each part carries six marks.

Q 1. (2X5=10)

a) Define sequence and limit point of a sequence with the help of an example.

b) State Sandwich theorem and give suitable example.

c) If $\lim_{x\to a} f(x) = l$ and $l \neq 0$, then prove that $\lim_{x\to a} \frac{1}{f(x)} = \frac{1}{l}$.

d) Show that sum and product of two continuous functions are also continuous.

e) Show that the function $f(x) = |x+1|$ is continuous but not derivable at $x = -1$.

f) Verify Lagrange's Mean Value Theorem for the function $f(x) = lx^2 + mx + n$ in $[a,\ b]$.

g) Find maxima and minima of the function $f(x) = \sin 2x + \frac{1}{3}\sin 3x$.

Q 2. (2X6=12)

a) State and prove Cauchy's 2nd theorem on limits

b) Find the limit of the given sequences

$(i)\ n^{-n-1}(n+1)^n,$ $(ii)\ \langle\frac{2n-7}{3n+2}\rangle.$

c) Find the reduction formula for $\int \frac{e^{mx}}{x^n}dx\ \ (n > 0)$.

Q3. (2X6=12)

a) Prove that if $\lim_{x\to a} f(x) = l$, then $\lim_{x\to a}|f(x)| = |l|$. But converse need not be true, give an example to justify.

b) Show that a function $f: \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ if and only if for each closed set H in $\mathbb{R}$, $f^{-1}(H)$ is also closed in $\mathbb{R}$.

c) Show that $\cos x$ is uniformly continuous on $[0,\ \infty[$, but $\cos x^2$ is not uniformly continuous on $[0,\ \infty[$.

Q 4. (2X6=12)

a) Prove that continuity is a necessary but not a sufficient condition for the existence of a finite derivative. Give example in the support.

b) Draw the graph of the function $y = |x-2| + |x+1| + |x+2|$ in the interval $[-4,\ 4]$ and discuss the continuity and differentiability of the function in this interval.

c) State and prove Rolle's Theorem. Also verify Rolle's Theorem for $f(x) = |x|$ in interval $[2, -2]$.

Q 5. (2X6=12)

a) Assuming the validity of the expansion, show that

(i) $\log \sec x = {x^2}/{2} + {x^4}/{12} + \cdots$.

(ii) $e^{\sin x} = 1 + x + {x^2}/{2} - {x^4}/{8} + \cdots$.

b) Use Taylor's Theorem to prove the following inequalities

$$0 \leq \sin x - \left(x - {x^3}/{3!} + {x^5}/{5!} - {x^7}/{7!}\right) \leq {x^9}/{9!}, \text{ for } x > 0.$$

c) Show that the function f, defined by $f(x) = \sin\left(x - {\pi}/{3}\right)$ for all $x \in \mathbb{R}$ has a minima at $x = {\pi}/{6}$ and maxima at $x = {\pi}/{2}$ and $x = -{\pi}/{6}$.

Q 6. (2X6=12)

a) Find all the asymptotes of the curve

$$f(x) = (x^2 - y^2)(2x + 3y)(3x + 2y) + 2x^2 + 3y^2 - 6$$

b) Find the radius of curvature of the catenary $x = c\log\left(s + \sqrt{c^2 + s^2}\right),\ y = \sqrt{c^2 + s^2}$.

c) Trace the curve $x = a(\theta - \sin\theta), y = a(a + \cos\theta),\ 0 \leq \theta \leq 2\pi$.

Department of Mathematics
Central University of
Haryana

Programme : Integrated B.Sc.-M.Sc.(Mathematics) Session : 2021-2022
Semester : I Max. Time : 3Hours
Course Title : Algebra Max. Marks: 70
Course Code : SBSMAT 03 01 02 C 5106

SECTION-I

Instructions:

1. Question no. 1 has has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Q.(1) a) If α and β be the imaginary cube roots of unity, prove that (4x3.5=14)

$$\alpha e^{\alpha x} + \beta e^{\beta x} = -e^{-x/2}\left[\sqrt{3}\sin\frac{\sqrt{3}}{2}x + \cos\frac{\sqrt{3}}{2}x\right]$$

b) Find a if the vectors $\begin{bmatrix}1\\-1\\3\end{bmatrix}, \begin{bmatrix}1\\2\\-3\end{bmatrix}, \begin{bmatrix}a\\0\\1\end{bmatrix}$ are linearly dependent.

c) Find the matrix representing the transformation $T: R^3 \rightarrow R^4$ defined by $T(x,y,z) = (x+y+z, 2x+z, 2y-z, 6y)$ relative to the standard basis of R^3 and R^4.

d) Find the general solution of $70x + 112y = 168$.

e) The relation of congruency is an equivalence relation.

f) Show that the elements on the main diagonal of a Hermitian matrix are all real.

g) Prove that the characteristic roots of a orthogonal matrix are of unit modulus.

SECTION-II

Q.(2) a) Define Countably Infinite set. Give an example of an infinite set but not Countably Infinite set .

b) If $f: A \rightarrow B$ and $g: B \rightarrow C$ be bijective functions, then gof is also bijective and $(gof)^{-1} = f^{-1}og^{-1}$.

c) Prove that $\left[\sin(\alpha+\theta) - e^{i\alpha}\sin\theta\right]^n = \sin^n\alpha\, e^{-in\theta}$ (2x7=14)

SECTION-III

Q.(3) a) State and prove fundamental theorem of arithmetic. (2x7=14)

b) Show that the congruence $x^3 + x^2 + 4x + 29 \equiv 0\ (mod 125)$ is satisfied by x = 109.

c) If $(a,m) = d$ and $d \mid b$, prove that the linear congruence $ax \equiv b (\text{mod } m)$ has exactly d solutions which are congruent mod m.

P.T.O.

SECTION-IV

Q.(4) a) Reduce to row reduced echelon form the matrix $A=\begin{bmatrix}1 & 1 & 2 & 3\\ 1 & 3 & 0 & 3\\ 1 & -2 & -3 & -3\\ 0 & 1 & 2 & 3\end{bmatrix}$ (2x7=14)

and also find rank of matrix A.

b) Solve the system of equations

$$4x+5y+6z=0$$
$$5x+6y+7z=0$$
$$7x+8y+9z=0$$

c) For what value of k (if any) the vector $v=(1,-2,k)$ can be expressed as a linear combination of vectors $v_1=(3,0,-2)$ and $v_2=(2,-1,-5)$ in $\boldsymbol{R}^3(\boldsymbol{R})$.

SECTION-V

Q.(5) a) Find the eigen vector of the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$. (2x7=14)

b) Determine whether the mapping $T:R^2\rightarrow R^3$ defined by $T(x,y)=(2x-y,x-y,-2x)$ is linear transformation or not?

c) The intersection of two subspaces W_1 and W_2 of a vector space $V(F)$ is also a subspace of $V(F)$.

CENTRAL UNIVERSITY OF HARYANA
First Semester Examinations 2023

Programme	: M.Sc. Mathematics	Session	: 2023-2024
Semester	: First	Max. Time	: 3 Hours
Course Title	: Real Analysis	Maximum Marks	: 70
Course Code	: SBSMAT 01 01 01 C 3104		

Instructions:
1. Question no. 1 has 7 sub parts and students need to answer any four. Each sub part carries three and half Marks.
2.Question number 2 to 5 have three sub parts and student need to answer any two. Each sub part carries 7 marks.

1. (a) Define complete order field. Give an example of order field which is not complete. (4x3.5=14)
   (b) Define discrete metric space. Is $d(x, y) = (x - y)^2$ is a metric or not.?
   (c) Show that a closed set contains all of it's limit points.
   (d) Is uniform convergence implies convergence justify your statement.
   (e) Define covering. Give an example of countable covering.
   (f) Show that $\cos x$ is a function of bounded variation over a finite interval.
   (g) Define Riemann sum and state the conditions under which Riemann integral exist.

2. (a) State and prove the Archimedean property. (2x7 =14)
   (b) State and prove the Bolzano-Weierstrass Theorem for sequences.
   (c) Show that a convergent sequence of real number is bounded. If $X = (x_n)$ and $Y = (y_n)$ be sequences of real number that converges to x and y respectively. Then show that the sequences $X + Y$ converges to $x + y$.

3. (a) Let $\{f_n)\}$ be a sequence of function defined on a set S. Then there exists f such that $f_n \longrightarrow f$ uniformly on S if and only if for each $\epsilon > 0$ there exists an N such that $m > N$ and $n > N$ implies $|f_m(x) - f_n(x)| < \epsilon$ for every $x \in S$. (2x7=14)
   (b) Define Riemann integrable. Show that the function $f : [1, 2] \longrightarrow \mathbb{R}$ defined by $f(x) = 3x+1, x \in [1, 2]$ is Riemann integrable.
   (c) Let f be strictly increasing function on a set $S \in \mathbb{R}$. Then show that f^-1 exists and is stricly increasing on $f(S)$.

4. (a) Let V be defined on $[a, b]$ as $V(x) = V_f(a, x)$, if $a < x \leq b$, $V(a) = 0$. Then show that V and $V - f$ are increasing function on $[a, b]$. (2x7=14)
   (b) Define directional derivative. Find the directional derivatives of $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$.
   (c) Define total variation. If f is continuous on $[a, b]$ and if f' exists and is bounded in the interior, say $|f'(x)| < A$ for all $x \in (a, b)$, then show that f is of bounded variation on $[a, b]$.

5. (a) State and prove Heine-Borel covering theorem. (2x7=14)
   (b) A subspace M of a complete metric space X is itself complete if and only if M is closed in X.
   (c) Define compactness. Show that a compact subset K of a metric space is closed and bounded.

Central University of Haryana

Term End Examination (Re-appear)

Programme: MSc (Mathematics) **Session: 2022-23**

Semester: I **Max. Time: 3 Hour**

Course Title: Differential Equations **Max. Marks: 70**

Course Code: SBSMAT 01 01 04 C 3104

Instructions:

1. *Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.*
2. *Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.*

Q1a. Find the general and singular solutions of: $3xy = 2px^2 - 2p^2$.

Q1b. Classify the following partial differential equations as hyperbolic/parabolic/elliptic and show all the working:

(i) $u_{xx} + u_{yy} = u_z$

(ii) $u_{xx} + 2u_{yy} + u_{zz} = 2u_{xy} + 2u_{yz}$

Q1c. Form a partial differential equation by eliminating the function f from $z = x^n f(y/x)$.

Q1d. Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible.

Q1e. Consider $f(x, y) = x^3 |y|$. Prove that f satisfies a Lipschitz condition on R: $|x| \le 2$, $|y| \le 2$ even though $\partial f/\partial y$ does not exist at (x, 0) if $x \neq 0$.

Q1f. Define the ordinary, regular and irregular singular points of the differential equation:

$$(x-1)\frac{d^2y}{dx^2} + (2x+3)\frac{dy}{dx} + 4xy = 0$$

Q1g. Give the definition of self-adjoint equation. Show that the Legendre's equation is self-adjoint.

Q2a. Apply Picard's method to find the solution of the initial value problem up to fourth approximation $dy/dx = y - x$, $y(0) = 2$.
Show that the iterative solution approaches the exact solution.

Q2b. Find the complete and singular solutions of $z=px + qy + p^2q^2$.

Q2c. State and prove Abel's formula.

Q3a. Show that x=0 is a regular singular point and hence solve in series the differential equation $x(x-1)y'' + (3x-1)y' + y = 0$.

Q3b. Find the characteristic values and characteristic functions of the Sturm-Liouville boundary value problem: $[xy']' + \left(\frac{\lambda}{x}\right)y = 0$; $y(1) = 0,\ y'(e^{2\pi}) = 0$, where the parameter λ is assumed to be non-negative.

Q3c. Solve $y_3 - 6y_2 + 11y_1 - 6y = e^{2x}$ by method of variation of parameters.

Q4a. Find the integral surface satisfying $4yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 1$, $x + z = 2$.

Q4b. Solve the PDE using Charpit's method

$$(p^2 + q^2)y = qz$$

Q4c. Solve the homogeneous partial differential equation

$(D^2 - 3DD' + 2D'^2)\, z = e^{2x-y} + \cos(x+2y)$

Q5a. A tightly stretched string of length ℓ with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3\left(\frac{\pi x}{\ell}\right)$. Formulate the problem mathematically and hence solve it for the displacement y(x,t).

Q5b. Reduce the one-dimensional wave equation $z_{xx} - z_{yy} = 0$ to canonical form.

Q5c. Solve $u_t = a^2 u_{xx}$ for the conduction of heat along a rod without radiation, subject to the following conditions: (i) u is not infinite for $t \to \infty$, (ii) $u_x = 0$ for x =0 and x=l.

CENTRAL UNIVERSITY OF HARYANA
Semester Examinations 2023

Programme	: M.Sc. Mathematics	Session	: 2023-2024
Semester	: Second	Max. Time	: 3 Hours
Course Title	: Topology	Maximum Marks	: 70
Course Code	: SBSMAT 01 02 02 C 3104		

Instructions:
1. Question no. 1 has 7 sub parts and students need to answer any four. Each sub part carries three and half Marks.
2.Question number 2 to 5 have three sub parts and student need to answer any two. Each sub part carries 7 marks.

1. (a) Define topological space. Give one example.
   (b) Show that a set is closed if and only if it contains all its limit points.
   (c) Let X, Y and Z be topological spaces. If $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ are continuous, then show that $gof : X \longrightarrow Z$ is continuous.
   (d) Write a short note on how Sierpinski Space is topologically different from Hausdorff space.
   (e) Define connected space. Show that $I = [0, 1]$ is connected.
   (f) Show that components are closed sets.
   (g) Show that every T_1 space is T_0 space but the converse is not true justify your statement.

2. (a) Show that every metric space is a topological space, but converse is not true justify your statement.
   (b) Let A be a subset of a space X. Then show that $\overline{A} = A \cup A'$ and $\overline{A} = A \cup \partial A$.
   (c) Let $\mathcal{B}$ be a collection of subsets of the set X such that $X = \cup\{B|B \in \mathcal{B}\}$ and for every two members B_1, B_2 of $\mathcal{B}$ and for each point $x \in B_1 \cap B_2$, there exists $B3 \in \mathcal{B}$ with $x \in B_3 \subseteq B_1 \cap B_2$. Then show that $\mathcal{B}$ is a basis for a topology on X.

3. (a) Define Product topology. Let $\{X_\alpha\}$ be an indexed family of spaces; let $A_\alpha \subset X_\alpha$ for each α. If ΠX_α is given either the product or the box topology then show that

$$\Pi \overline{A}_\alpha = \overline{\Pi A_\alpha}$$

   (b) Define homeomorphism. State and prove the Pasting Lemma.
   (c) State and prove the Tychonoff Theorem.

4. (a) Show that a path connected space is connected but converse is not true, justify your statement.
   (b) Prove that a connected and locally connected space is path connected
   (c) Prove that a continuous image of a compact space is compact.

5. (a) Prove that a non-empty product space is a T_2 space if and only if each factor space is a T_2 space.
   (b) Define regular space. Show that every metric space is a T_3 space. Give one example to show that a T_2 space need not be a T_3 space.
   (c) A topological space X is normal if and only if given a closed set A in X and a continuous map, $f : A \longrightarrow \mathbb{R}$, there is a continuous function $F : X \longrightarrow \mathbb{R}$ such that $F|_A = f$.